A-Level Maths – Induction Summer Work

Name:_____.

(Show workings for every question in this booklet)

This booklet contains GCSE Algebra skills that you will need in order to successfully complete the A-Level Maths course. In addition to this booklet you should also be confident in the following areas;

- Trigonometry
- Sine and Cosine Rule
- Indices and Surds
- Equations of straight lines

Resources to help you;
MyMaths is a great resource for A-Level students, especially look at the A/A* Booster packs

Bridging GCSE and A Level Maths (Mark Rowland)
ISBN number; 978-0-00-743171-7 is a good book to look at if you need some extra support.

CGP Mind the Gap from www.cgpbooks.com (similar to the practice papers you had at GCSE)

Review of skills needed to successfully complete C1 exam paper.

Expanding Brackets:

Remove the brackets and simplify (multiply out the brackets):

(x + 2) (x + 3)

$(x + 2)(x + 3)$ = $x^2 + 2x + 3x + 6$ = **$x^2 + 5x + 6$**

Remember:

(i) Not to try and add x^2 and 5x together they are not similar and cannot be collected.

(ii) $(x + 5)^2$ = $(x + 5)(x + 5)$ = **$x^2 + 5x + 5x + 25$**
= **$x^2 + 10x + 25$**

Questions - Remove the brackets and simplify the following, (careful with signs):

1) $(x + 1)(x + 3)$

2) $(x + 4)(x + 5)$

3) $(x + 5)(x - 2)$

4) $(x - 7)(x + 5)$

5) $(x - 3)(x + 3)$

6) $(2x + 1)(x - 3)$

7) $(2y - 3)(y + 1)$

8) $(y + 4)^2$

9) $(a - 2)^2$

10) $(x + 1)^2 + (x + 2)^2$

Factorising using a single bracket:

Factorise: **$x^2 + 7x$**

Find the value or values that are common (the same) in all terms in the expression needing factorising. In this case there is a single **x** in both terms; the **x** is taken outside a single bracket. Inside the brackets are the terms that need to be multiplied with **x** to create the original value.

x (x + 7)

Check your answer by removing the brackets, as before, making sure you end up with the original expression.

Factorise: **$6a^2b - 10ab^2$**

The common values in this instance are, **2**, **a** and **b**. The value **2ab** is taken outside a single bracket. Inside the bracket are the terms that need to be multiplied with **2ab** to create the original value.

2ab (3a – 5b)

Remember when you see the value **2ab** it is **2** multiplied by **a** multiplied by **b**.

Factorise the following completely:

1) $2x^2 + 3y$
2) $3x^2 - 21x$
3) $6c^2 - 21c$
4) $56y - 21y^2$
5) $x^2 + xy + 3xz$
6) $3a^2 b + 2ab^2$
7) $2kx + 6ky + 4kz$
8) $x^2 k + xk^2$
9) $abc - 3b^2 c$
10) $2abx + 2ab^2 + 2a^2 b$

Factorising using double brackets (quadratic equations):

Factorise: **$x^2 + 6x + 8$**

IV. Find two numbers that multiply together give 8 and add together to give 6.
IV. Put these numbers into double brackets.

Numbers that multiply together to give 8: 1 x 8 = 8
2 x 4 = 8

The same sets of numbers added give: 1 + 8 = 9
2 + 4 = 6

The numbers included in the brackets are therefore **2** and **4**. This works because the coefficient of x^2 is 1, when the coefficient is not 1 readjustment to this technique is needed.

$x^2 + 6x + 8$ = (x + 2) (x + 4)

Factorise the following, being careful of the signs used within the brackets:

1) $x^2 + 7x + 10$
2) $x^2 + 10x + 21$
3) $y^2 + 11y + 24$
4) $a^2 - 3a - 10$
5) $x^2 - 2x - 35$
6) $y^2 - 5y + 6$
7) $a^2 + 14 + 45$
8) $y^2 + 2y + 1$
9) $x^2 - 8x - 240$
10) $x^2 - 49$
11) $x^2 - 9$
12) $x^2 - 16$

Methods of solving quadratic equations:

(Expressions contain no equals signs, equations do.)

(i) Using factorising to solve equations

Solve the equation $\mathbf{x^2 + x - 12 = 0}$

Factorise the quadratic $\mathbf{(x + 4)(x - 3) = 0}$

The only way it is possible to multiply two values together and get the answer to be equal to zero is if one (or both) of the values is zero.

This allows us to make the following conclusions.

Either $\mathbf{(x + 4) = 0}$ OR $\mathbf{(x - 3) = 0}$

$\mathbf{\underline{x = -4}}$ $\mathbf{\underline{x = 3}}$

In the same way solve the following equations:

1) $x^2 + 7x + 12 = 0$

2) $x^2 + 2x - 15 = 0$

3) $x^2 - 5x + 6 = 0$

4) $x^2 + 5x - 14 = 0$

5) $x^2 + 10x + 21 = 0$

6) $x^2 - 4x - 5 = 0$

(ii) Solution by formula

The solution of the quadratic equation $\mathbf{ax^2 + bx + c = 0}$ are given by the formula:

$$\mathbf{x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}}$$

This formula should only be used if you are unable to factorise (you may loose marks on the exam paper for using this instead of factorising).

Solve the equation $\mathbf{2x - 3x - 4 = 0}$ unable to factorise

In this case a = 2, b = - 3, c = - 4.

Substitute into the equation $\mathbf{x = \frac{-(-3) \pm \sqrt{[(-3)^2 - (4 \times 2 \times -4)}}{2 \times 2}}$

Careful with the signs $\mathbf{\frac{x = 3 \pm \sqrt{(9 + 32)}}{4} = \frac{3 \pm \sqrt{41}}{4}}$

As the C1 paper is non – calculator the answer can be left in surd form unless otherwise stated.

Solve the following giving answers in surd form:

1) $2x^2 + 11x + 5 = 0$

2) $6x^2 + 7x + 2 = 0$

3) $5x^2 - 7x + 2 = 0$

4) $2 - x - 6x^2 = 0$

Factorising where the squared coefficient is not 1

Factorise **$3x^2 + 13x + 4$**

I. Find two numbers which multiply to give (3 x 4), i.e. 12, and add up to 13 in this case the numbers are **1** and **12**.
II. Split the '13x' term into these number of x, **$3x^2 + x + 12x + 4$**
III. Factorise in pairs, **$3x^2 + x = x(3x + 1)$**
$12x + 4 = 4(3x + 1)$

$3x^2 + x + 12x + 4$ = **$x(3x + 1) + 4(3x + 1)$**

IV. **(3x + 1)** is common, therefore

$3x^2 + x + 12x + 4$ = **(3x + 1) (x + 4)**

Factorise: **$2x^2 + 5x + 3$**

I. Find two numbers which multiply to give (2 x 3), i.e. 6, and add up to 5 in this case the numbers are **2** and **3**.
II. Split the '5x' term into these number of x, **$2x^2 + 2x + 3x + 3$**
III. Factorise in pairs, **$2x^2 + 2x = 2x(x + 1)$**
$3x + 3 = 3(x + 1)$

$2x^2 + 2x + 3x + 3$ = **$2x(x + 1) + 3(x + 1)$**

IV. **(x + 1)** is common, therefore

$2x^2 + 2x + 3x + 3$ = **(x + 1) (2x + 3)**

Factorise the following, being careful of the signs used within the brackets:

1) $2x^2 + 11x\ 12$

2) $3x^2 – 5x – 2$

3) $3x^2 – 17x – 28$

4) $6x^2 – 19x + 3$

More solving equations by factorising

Solve the following equations by factorising (remember to make the equation equal to zero):

1) $3x^2 + 10x - 8 = 0$

2) $2x^2 + 7x - 15 = 0$

3) $6x^2 - 13x + 6 = 0$

4) $10x^2 - x - 3 = 0$

5) $12y^2 - 16y + 5 = 0$

6) $6a^2 - a - 1 = 0$

7) $2x^2 + 7x = 15$

8) $4x^2 - 29x = -7$

Changing the subject of the formula

The operations required when solving linear equations are exactly the same as the operations required in changing the subject of a formula.

Make **x** the subject of the formula	M**x** + B = A
Rearrange getting the M**x** on one side	M**x** = A – B
Divide by M on both sides to get **x** alone	$\mathbf{x = \frac{A - B}{M}}$
Make **y** the subject of the formula	x(**y** – a) = e
Remove the brackets	x**y** – ax = e
Rearrange getting the x**y** on one side	x**y** = e + ax
Divide by x on both sides to get **y** alone	$\mathbf{y = \frac{e + ax}{x}}$

Make **x** the subject of the following:

1) A**x** = B
2) N**x** = T
3) 9**x** = Y + N
4) $R - S^2 = N\mathbf{x}$
5) $N^2 + \mathbf{x} = T$
6) F = **x** – B
7) Dx + E = F
8) 3(x – 1) = 5
9) D(x + E) = F

Make a the subject of the following

1) $\frac{a}{4} = 3$

2) $\frac{a}{N} = R$

3) $\frac{a - D}{N} = A$

4) $\frac{Aa + B}{C} = D$

5) $\frac{M + Aa}{b} = c$

6) $6 - a = 2$

7) $C - a = E$

8) $b = S - a$

9) $T - Xa = B$

10) $R = v^2 - ra$

11) $\frac{5 - 7a}{3} = 2$

12) $\frac{y(x - a)}{z} = t$

Make a the subject of the following:

1) $\frac{7}{a} = 14$

2) $t = \frac{v}{a}$

3) $\frac{v}{a} = \frac{m}{s}$

4) $\frac{V}{A - T} = D$

5) $T = \frac{b}{c - a}$

Make x the subject of the following:

1) $\frac{2}{x} + 1 = 3$

2) $\frac{r}{x} - t = n$

3) $C - \frac{d}{x} = e$

4) $3M = M + \frac{N}{P + x}$

5) $A = \frac{B}{C + x} - 5A$